UCH O'LCHOVI DIFFERENSIAL TENGLAMALAR SISTEMASINING MUVOZANAT NUQTALARINING TURLARI

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Annatasiya. *Uch o'lchovli fazoda o'zgarmas koeffsentli bir jinsli differensial tenlamalar sistemasini trayektoriyalarini muvozanat nuqta* $O(0{,}0{,}0)$ *atrofida joylashishini o'rganiladi.*

Kalit so'zi: *Muvozanat, xaraktrestik tenglama, asimtotik, tugun, turg'unmas, trayektoriya, turg'un tugun, Turvis sharti, turg'un fokus, turg'unmas fokus, focus markaz, karrali haqiqiy va har xil, qo'shma kompleks turg'unmas egar, turg'un tug'ma tugun.*

Bu maqolada o'zgarmas koeffesintli bir jinsli tenglamalar sistemasi uch o'lchovli fazoda berilgan bo'lsin

$$\begin{cases} \dfrac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ \dfrac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ \dfrac{dx_3}{dt} = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{cases} \qquad detA \neq 0, \qquad A = (a_{ij}) \quad (1)$$

Sistema traektoriyalarning $O(0{,}0{,}0)$ muvazanat (yoki maxsus) nuqta atrofida joylashini o'rganamiz (1) Sistema yechimini $x_k = \alpha e^{2t}$ $(k = \overline{1,3})$ ko'rinishida izlaymiz λ ni topish uchun xarakteristik tenglama tuzamiz

$$\Delta(\lambda) = \begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0 \qquad (2)$$

Yoki $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$ $\alpha_1,\ \alpha_2,\ \alpha_3$- lar aniq ko'paytuvchilarga ikkitasi uchinchisi orqali quydagi sistemadan topiladi.

$$\begin{cases} (a_{11}-\lambda)\alpha_1 + a_{12}\alpha_2 + a_{13}\alpha_3 = 0 \\ a_{21}\,\alpha_1 + (a_{22}-\lambda)\alpha_2 + a_{23}\alpha_3 = 0 \\ a_{31}\alpha_1 + a_{32}\alpha_2 + (a_{33}-\lambda)\alpha_3 = 0 \end{cases} \qquad (3)$$

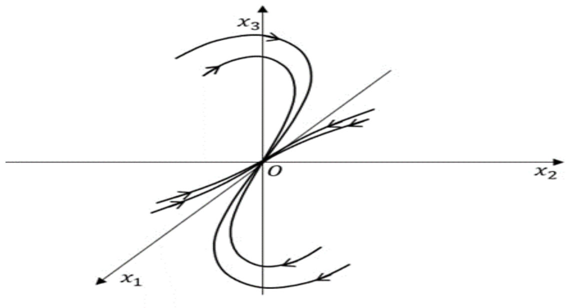
Quydagi hollarda bo'lishi mumkin ularni ko'rib chiqamiz alohida-alohida

a) Xarakteristik tenglamani ildizlari λ_i $(i = \overline{1,3})$ haqiqiy va har xil, u holda uning yechimi $x_k = c_1\beta_k e^{\lambda_1 t} + c_2\gamma_k e^{\lambda_2 t} + c_3\delta_k e^{\lambda_3 t}$ $\quad k = \overline{1,3}$ (4) ko'rinishda yoziladi.

$\beta_k, \gamma_k, \delta_k$ lar (3) sistemaga mos ravishda $\lambda = \lambda_1, \lambda = \lambda_2, \lambda = \lambda_3$ larni quydagilar orqali topiladi c_1, c_2, c_3 lar erkli o'zgarmas. O'z navbatida bu yerda quydagi holler bo'lishi mumkin.

1) barcha i lar uchun $\lambda_i < 0$. U vaqtda muvozanat nuqta $x_1 = 0, x_2 = 0, x_3 = 0$ asimtotik to'rg'un, chunki (3) formuladagi ko'paytuvchi $e^{\lambda_i t}\ lar\ t = t_0$ moment koordinatalari boshini δ atrofida bo'lgan barcha nuqtalar t ning istalga kata qiymatlari uchun koordinatalar boshining

istalgan kichik δ atrofiga tushib qoladi va $t \to \infty$ da koordinatalar boshiga intiladi. 1-chizmada muvozanat nuqta atrofidagi traektoriyalar joylashishi ko'rsatilgan. Muvozanat turg'un, tugun turg'un deyladi. Strelkalar t o'sishi bilan traektoriyalar bo'yicha harakat yo'nalishini bildiradi. Bu hol (1) Sistema quydagi koefsentli tengsizliklar bajarilishi o'rinli bo'ladi.



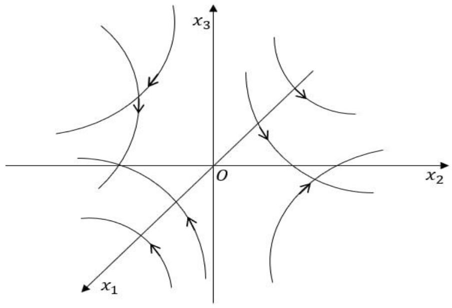
1- chizma

$$a_1 > 0, \qquad \Delta_2 = \begin{vmatrix} a_1 & 1 \\ a_3 & a_2 \end{vmatrix} > 0, \qquad a_3 > 0, D < 0.$$

Bu yerda $D = q^2 + p^2$ xarakteristik tenglamaning diskriminanti

$$2q = \frac{2a_1^3}{27} - \frac{a_1 a_2}{3} + a_3, \;\; 3p = \frac{3a_2 - a_1^2}{3}$$

2) $\lambda_1 > 0, \lambda_2 > 0, \lambda_3 > 0$ bo'lsin. Bu holda t ni t ga olmashtersak, yuqorida ko'rilgan strelkalarini teskari quyish kerak) muvozanat nuqta turg'unmas tugun deyladi bu hol koefsentli $a_1 < 0, \Delta_2 < 0,\; a_2 > 0, D < 0$ tengsizliklar bajarilganda o'rinli bo'ladi.



2- chizma

3) Agar $\lambda_1 > 0, \lambda_2 < 0, \lambda_3 < 0$ bo'lsa, muvozanat nuqta turg'unmas chunki

$$x_k = c_1 \beta_k e^{\lambda_i t}$$

Traektoriyalar bo'yicha harakat qiluvchi nuqtalar (c_1 ni qanchalik kichik olmaylik) t o'sishi bilan koordinatalar boshidagi ε atrofidan chiqadi $t \to \infty$ koordinatalar boshiga intiluvchi harakatlar $x_k = c_2 \gamma_k e^{\lambda_2 t} + c_3 \delta_k e^{\lambda_3 t}$ ko'rinishda bo'lib, butun bir tekslikni tashkil qiladi (2-chizma).

(1) sistema $D < 0$ gurbis sharti bajarilgan holda uchraydi

b) Xarakteristik tenglamani ildizlari bajarilmagan holda uchraydi ikkitasi qo'shma kompleks, ya'ni $\lambda = \lambda_1, \lambda_{2,3} = p \pm qi$, $q \neq 0$ bu holda Sistema uchun umumiy yechim

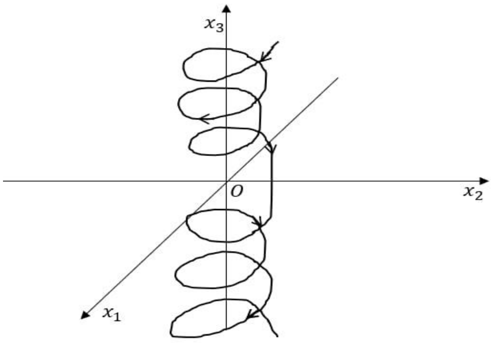
$$x_k = c_1 \beta_k e^{\lambda_1 t} + e^{pt}(c_1 cosqt + c_2 sinqt) \;\; k = \overline{1,3} \qquad (5)$$

ko'rinishda bo'ladi $c_2^i > c_3^i\; lar\; (i = 2,3)$ c_2^i, c_3^i larning birorta chiziqli konbinatsiyasidan iborat bo'ladi. Shu sabali bu yerda quydagi holler uchraydi:

1) $\lambda_1 < 0, \lambda_{2,3} = p \pm qi$, $p < 0$ $q \neq 0$ ko'paytuvchi e^{pt}, $p < 0$ $t\, ning$ o'sishi bilan nolga intiladi, ikkinchi davriy ko'paytuvchi (5) formulada har doim chekli bo'ladi. $t = t_0$

momentda koordinatalar momentda koordinatalar boshininh δ atrofida bo'lgan nuqta traektoriya buylab harakat qilib t ning o'sishi bilan koordinatalar boshiga yaqinlashib boradi va $t \to \infty$ da nolga intiladi. Nol yechim- asimtotik turg;un muvozanat nuqtaga ega turg'un focus deyladi (3-chizma).

Bu holda $D > 0,\ \ a_1 > 0,\ \ \Delta_2 > 0,\ \ a_3 > 0$ bo'ladi.



3-chizma

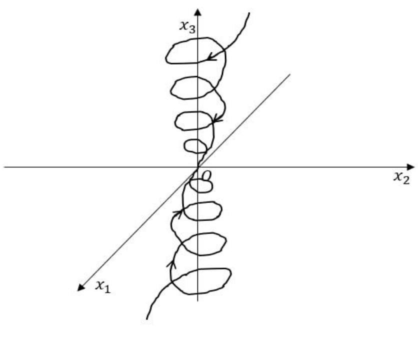
2) $\lambda_1 > 0, \lambda_{2,3} = p \pm qi\ ,\ \ p > 0\ \ q \neq 0$ bo'lsin bu yerda $t\ ni\ t\ ga$ almashtersak qolgan hollarga kelamiz. Traektoriyalar joylashishi 3-chizmadagi kabi bo'ladi, lekin stelkalarteskari quyiladi. Muvozanat nuqta turg'unmas focus deyladi.

Bu holda $D > 0,\ \ a_1 < 0,\ \ \Delta_2 < 0,\ \ a_3 > 0$ bo'ladi

3) Agar $\lambda = \lambda_1, \lambda_{2,3} = p \pm qi\ ,\ \ p < 0,\ \lambda,\ q \neq 0$ bo'lsa (5) formuladagi qushuluvchilardan bittasi o'suvchi, hisoblanadi $t = t_0\ momenta\ \delta$ atrofida bo'lgan t o'sishi bilan traektoriya bo'ylab koordinata boshidan uzoqlasha boshlaydi, demak nol yechim turg'unmas. Muvozanat nuqtaga egar-fokus deyladi. $D > 0$ bo'lib, Turvis sharti bajarilmasa muvozanat nuqta egar-fokus bo'ladi.

4) $\lambda_1, \lambda_{2,3} = \pm qi\ ,\ \ p = 0$ bo'lsa λ_1 ning ishorasiga qarab nol yechim turg'un yoki turg'unmas bo'lishi mumkin.

Muvozanat nuqta focus markaz (4-chizma) egar-fokus bo'lishi mumkin.



4-chizma

b) Xarakteristik ildizlari karrali $\lambda_1 = \lambda_2,\ \lambda_3,\ \lambda_1 \neq \lambda_3$ ya'ni xarakteristik tenglamani diskeriminanti $D = 0$. Bu holda (1) sistemaning umumiy yechimi

$$x_k(t) = (c_1\beta_k + c_2\gamma_k t)e^{\lambda_1 t} + c_3\delta_k e^{\lambda_3 t},\quad k = \overline{1,3} \qquad (6)$$

Kurinishda bo'lib kupaytuvchi $e^{\lambda_1 k}, t \to \infty$ da nolga intiladi. Shu bilan birga koordinatalar boshining δ- atrofidagi istalgan nuqta t o'sishi bilan ε- atrofiga tushadi va nolga intiladi. Nol yechim asimtotik turgun bo'ladi. Muvozanat nuqta turg'un deyladi bu holda muvozanat a), 1) bilan turg'un focus b), 1) orasidagi chigaraviy dolda mos keladi, chunki a_{ij} parametrlari ozroq o’zgartirilganda muvozanat turg’un tugun, a), 1) holda yoki turg’un fokudga o’tishi mumkin, ya’ni a_{ij} parametriga o’zgarish berilganda xaraktrestik tenglamaning karrali ildizlari oddiy haqiqiy yoki qushma kompleks ildizlari ajralishi mumkin.

2) Agar $\lambda_1 = \lambda_2 > 0,\ \lambda_3 > 0\ bo'lsa\ t\ ni\ tga$ almashterilsin 1) holdagi natijaga kelamiz. Faqat traektoriyalar buyicha harakat teskari yo’nalishda bo’ladi. Muvozanat nuqta turg’unmas tugun deyladi.

3) $\lambda_1 = \lambda_2 > 0,\ \lambda_1 \cdot \lambda_3 < 0$ bo’lsin. Nol yechim tugunmas bo’ladi,chunki (6) formulada $t \to \infty\ da\ \|x(t)\| \to \infty$. Muvozanat nuqta egar deyladi, lekin bu egar

a), 3) holdagi egardan farqi shundaki a_{ij} parametrlarni o’zgarterganda a), 3) holdagi egar yoki egar fokus b), 3) dagi tugunga o’tish mumkin, chunki a_{ij} paramertlarning o’zgarishi natijasida ajratilishi mumkin.

a) $\lambda_1 = \lambda_2 = \lambda_3$ bo’lsin. Xaraktrestik tenglamaning diskerminantida $D = 0$, (1) Sistema uchun $x_k(t) = (c_1\beta_k + c_2\gamma_k t + c_3\delta_k t^2)e^{\lambda_1 t},\quad k = \overline{1,3}$ (7)

ko’rinishda yoziladi. Quydagi hollar bo’lishi mumkin .

1) $\lambda_1 < 0$ bo’lsin $e^{\lambda_1 t}$ ko’paytuvchi $t \to \infty$ da tezroq nolga va shu sababli $(c_1\beta_k + c_2\gamma_k t + c_3\delta_k t^2)e^{\lambda_1 t}$ ifoda $t \to \infty$ da nolga intiladi. Nol yechim asimtotik turg’un. Muvozanat nuqta turg’un tugun boladi.

2) $\lambda_2 > 0$ bo’lsin t ni t ga almashtersak, 1) holdagi natijaga kelamiz. Traektoriyalar buyicha harakat teskari yunalishda bo’ladi. Muvozanat nuqta turg’unmas tugun bo’ladi. Xaraktristik sonlar noldan farqli bo’lgan barcha hollarni qarab chiqdik, chunki shartga ko’ra $detA \neq 0$. Demak yo’qoridagilarga asoslanib tekslikda muvozanat nuqtani quydagicha sinflash mumkin ekan.

a) $\lambda_1 va\ \lambda_3$ haqiqiy va $\lambda_1 \neq \lambda_3$. Bu holda muvozanat nuqta;

1) $\lambda_1 < 0\ ,\quad \lambda_3 < 0$ turg’un tugun;

2) $\lambda_2 > 0,\quad \lambda_3 > 0$ turg’unmas tugun;

3) $\lambda_2 < 0,\quad \lambda_3 > 0$ turg’unmas egar;

b) $\lambda_{2,3} = q \pm qi$. Bu holda;

1) $p < 0$ turg’un fokus;

2) $p > 0$ turg’unmas focus;

3) $p \neq 0$ markaz tugun;

d) $\lambda_1 = \lambda_3$. Bu holda;

1) $\lambda_2 = \lambda_3 < 0$ turg’un tug’ma tugun;

2) $\lambda_2 = \lambda_3 > 0$ turg’unmas tug’ma tugun;

Agar $detA = 0$ bo’lgan holda (1) tenglamalar sistemasining muvozanat nuqtalarini kelgusi ishlarimizda sinflaymiz.

Xulosa

Xulosa qilib aytganda bir jinsli uch o’lchovli differensial tenglamalar sistemasini yechimlarinin muvozanat nuqta $O(0{,}0{,}0)$ atrofida sinflash uchun tenglamalar sistemasini xaraktrestik tenglamasini yechimlari $\lambda_1,\ \lambda_2,\ \lambda_3$ larning qiymatlariga qarab

$detA \neq 0$ da bturg’un tugun, egar focus va turg’unmas tugun , egar focus, markaz ekanligini teksheriladi.

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