THE EFFECT OF THE HEAT SOURCE ON THE AMBIENT DENSITY IN THE PROCESSES OF NON-LINEAR HEAT PROPAGATION IN MULTIDIMENSIONAL FIELDS

J.M.Toshtemirov
University of Science and Technologies (Tashkent, Uzbekistan)
jaloliddintoshtemirov0@gmail.com
M.A.Asadullayeva
University of Science and Technologies (Tashkent, Uzbekistan)
lyudaasadullayeva01@gmail.com
https://doi.org/10.5281/zenodo.10052972

Abstract. *In nonlinear heat propagation processes in the article in multidimensional fields devoted to the study of the effect of the heat source on the density of the environment.*

In the course of work, a self-similar solution was built to solve the linear heat diffusion equation according to the characteristics of the environment density and heat source, reaction-diffusion processes were observed, and theorems were proved. The following results were obtained from this work: a preliminary estimate for the double linear heat diffusion equation, the localization process was observed, the finite velocity was approximated, new effects were observed, and an algorithm based on the obtained self-similarity was constructed. solution, the program code was created in the programming language and the process was modeled. All results were compared.

Keywords: *Reaction-diffusion, parabolic equation, asymptotic, finite speed, approximation, conductivity of convective migration, self-similar, Multidimensional areas, system, numerical solution.*

ВЛИЯНИЕ ИСТОЧНИКА ТЕПЛА НА ПЛОТНОСТЬ СРЕДЫ В ПРОЦЕССАХ НЕЛИНЕЙНОГО РАСПРОСТРАНЕНИЯ ТЕПЛА В МНОГОМЕРНЫХ ПОЛЯХ

Аннотация. *В нелинейных процессах распространения тепла статья в многомерных областях посвящена изучению влияния источника тепла на плотность среды.*

В ходе работы было построено автомодельное решение для решения линейного уравнения диффузии тепла по характеристикам плотности среды и источника тепла, наблюдались реакционно-диффузионные процессы и доказаны теоремы. В результате работы были получены следующие результаты: проведена предварительная оценка для двойного линейного уравнения теплодиффузии, обнаружен процесс локализации, аппроксимирована конечная скорость, обнаружены новые эффекты и построен алгоритм на основе полученного самоподобия. Для решения задачи был создан программный код на языке программирования и смоделирован процесс. Все результаты сравнивались.

Ключевые слова: *Реакция-диффузия, параболическое уравнение, асимптотика, конечная скорость, аппроксимация, проводимость конвективной миграции, автомодельность, Многомерные области, система, численное решение.*

Introduction

At present, one of the most frequent processes of quenching is the process of heat dissipation, when the main phenomenon in this process depends on what area the heat is distributed. The philosophy of Life shows that the spread of heat in nature can be either in a multidimensional area or in a one-dimensional area. Of course, in this process, we must draw up the evolutionary equation of the process, taking into account the density of the environment, the positive or negative of the heat source, the heat capacity, the dependence of the environment on the coefficient of heat transfer. As we know, the processes of heat dissipation in multidimensional areas are in a non-linear state. In such a process, the main attention is paid to heat capacity. For the process of heat dissipation without any scratches, the initial temperature (Cauchy condition) must be given. For non-linear heat dissipation, quasi-linear diffusion models, the spatial localization property and the thermal dissipation effect with limited speed have been studied by many scientists over the years. It was also used in nonlinear processes encountered in multidimensional areas of the studied mathematical models and found new effects not characteristic of linear equations. The scientific research devoted to nonlinear systems for cases of varying density, conductibility capacity of the environment, conductibility of convective migration, which leads to parabolic equations with distortion, has begun to be viewed as the main problems of the present day. In order to impress the process of heat dissipation without lines in multidimensional areas, we see the following equation in $Q=\{(t,x): t\in R_+, x\in R\}$ area:

$$Au \equiv -\rho_1(|x|)u_t + \nabla\left(\rho_2(|x|)u^{m-1}|\nabla u^k|^{p-2}\nabla u^l\right) + v\nabla u + \varepsilon\gamma u^\beta \tag{1}$$

initial (Cauchy condition)

$$u(0,x)=u_0(x)\geq 0, \tag{2}$$

let condition given.

Here $u(x,t)$-the temperature of the heat, $v(t)\in c(Q)$-the speed of the environment, $\rho_1(|x|)=|x|^{-n}, \rho_2(|x|)=|x|^q$-the density of the environment, which is a continuous function β-parameter, γu^β- represents the positive ($\varepsilon=+1$) or negative ($\varepsilon=-1$) power of the source of heat.

(1) the equation represents a number of physical processes: the reaction diffusion process in a non-linear environment, the heat dissipation process in a non-linear environment, the filtration of liquid and gas in a non-linear environment, they represent the existence of the law of polytropy and other non-linear displacements.

(1) the Cauchy issue and boundary value issues for the equation were observed by many authors in one-dimensional and multi-dimensional cases [1-5].

(1) in the processes represented by the equation, the phenomenon of finite distribution of temperature occurs [4]. In the presence of an absorption coefficient, the phenomenon of the “rear” front can occur, that is, the Left front can stop after a certain time and move along the medium.

Solution method

There are many ways to find a solution to the above (1) equation: in functional analysis, it is possible to solve with the help of spaces, in mathematical physics and differential equations, to build differential operators, and in practical mathematics, mainly by methods of constructing an self-similar solution.

(1) we write the equation for N=2 case as follows:

$$\begin{cases} |x|^{-n}\dfrac{\partial u_1}{\partial t} = \nabla\left(|x|^q u_i^{m_1-1}|\nabla u_1^k|^{p-2}\nabla u_1^l\right) + v(t)\nabla u_1 + \gamma_1 u_2^{\beta_1} \\ |x|^{-n}\dfrac{\partial u_2}{\partial t} = \nabla\left(|x|^q u_i^{m_2-1}|\nabla u_2^k|^{p-2}\nabla u_2^l\right) + v(t)\nabla u_1 + \gamma_i u_1^{\beta_2} \end{cases} \tag{3}$$

here, $\beta_i \neq -1; \beta_i\cdot\beta_{3-i}\neq 1$, $\gamma_i u_i^\beta$ - the source of heat,

$u_i(t,|x|)|_{t=0}=u_{i0}(|x|)\geq 0, \quad x\in R^N$, $i=1,2$ - the balance of conditions.

(3) system of equations to be present in solution in $m=k, n=0, l=1, 0<\beta_i<1$

position[3] $\beta_i = \dfrac{p-[m_i\cdot(p-1)-1]}{p-1}, 1<m_i<2, \; p>m_i\cdot(p-1)-1$

(3) in the system of equations, we perform the following substitution

$$|x| = r = \left(\sum_{i=1}^{N} x_i^2\right)^{1/2};$$

here is the size of the space. This replacement is called a radial-symmetrical replacement.
From the above substitution (3) the system of equations changes as follows:

$$\begin{cases} r^{-n}\dfrac{\partial u}{\partial t} = r^{1-N}\dfrac{\partial}{\partial r}\left(r^{q+N-1}u_1^{m_1-1}\left|\dfrac{\partial u_1^k}{\partial r}\right|^{p-2}\dfrac{\partial u_1^l}{\partial r}\right) + r^{1-N}v(t)\dfrac{\partial}{\partial r}\left(r^{N-1}\dfrac{\partial u_1}{\partial r}\right) + \gamma_1 u_2^{\beta_1} \\ r^{-n}\dfrac{\partial u}{\partial t} = r^{1-N}\dfrac{\partial}{\partial r}\left(r^{q+N-1}u_2^{m_2-1}\left|\dfrac{\partial u_2^k}{\partial r}\right|^{p-2}\dfrac{\partial u_2^l}{\partial r}\right) + r^{1-N}v(t)\dfrac{\partial}{\partial r}\left(r^{N-1}\dfrac{\partial u_2}{\partial r}\right) + \gamma_2 u_1^{\beta_2} \end{cases} \quad (4)$$

(4) in order to solve the system of equations, we initially look for the solution as follows:

$$u_i(t,r) = v^{\alpha_i}\cdot w_i(\xi);\ \xi = r\cdot v^{-\mu};\ i = 1,2 \quad (5)$$

here α_i - the number being searched, w_i - an unknown function, ξ - parameter, $\mu = const.$
(5) provisions (1) we apply to the system of equations:

$$r^{-n}\frac{\partial u_i}{\partial t} = \xi^{-n}v^{-n\mu}\left[\alpha_i v^{\alpha_i-1}v' w_i - \mu v^{\alpha_i-1}v'\xi w_{i\xi}\right] = \xi^{-n}v^{\alpha_i-1-n\mu}v'\left[\alpha_i w_i - \eta\xi w_{i\xi}\right];\ i = 1,2$$

$$r^{-n}\frac{\partial}{\partial r}\left(r^{q+N-1}u_i^{m_i-1}\left|\frac{\partial u_i^k}{\partial r}\right|^{p-2}\frac{\partial u_i^k}{\partial r}\right) = \xi^{1-N}v^{\mu(p-q)+\alpha_i(k(p-2)+l+m_i-1)}\frac{d}{d\xi}\left(\xi^{q+N-1}w_i^{m_i-1}\left|\frac{dw_i^k}{d\xi}\right|^{p-2}\frac{dw_i^l}{d\xi}\right)$$

$$r^{-n}v\frac{\partial}{\partial r}\left(r^{N-1}\frac{\partial u_i}{\partial r}\right) = \xi^{1-N}v^{1+\alpha_i-2\mu}\frac{d}{d\xi}\left(\xi^{N-1}\frac{dw_i}{d\xi}\right)$$

$$\gamma_i u_{3-i}^{\beta_i} = \gamma_i v^{\alpha_{3-i}\beta_i}w_{3-i}^{\beta_i}$$

v -we set the following conditions for the function:

$$v'v^{\alpha_i-1-n\mu} = v^{\alpha_{3-i}\beta_i}$$

$$v^{-n\mu+\alpha_i-\alpha_{3-i}\beta_{i-1}}dv = dt$$

$$v(t) = A_0(T_0+t)^{\frac{1}{\lambda_0}}$$

it is here $\lambda_0 = \alpha_i - \alpha_{3-i}\beta_i - n\mu \neq 0;\ A_0 = (\lambda_0)^{\frac{1}{\lambda_0}}$. If: $\lambda_0 = 0$, $v(t) = c_1 t:\ c_1 = const$

As a result, we will have a new equality

$$\mu(q-p) + \alpha_i(m_i + k(p-2) + l - 1) = 1 + \alpha_i - 2\mu = \alpha_{3-i}\beta_i \quad (6)$$

(6) from the equation, the following system of linear equations is formed:

$$\begin{cases} \alpha_1(m_1 + k(p-2) + l - 2) + \mu(q-p+2) = 1 \\ -\alpha_1 + \alpha_2\beta_1 + 2\mu = 1 \\ \alpha_1\beta_2 - \alpha_2 + 2\mu = 1 \end{cases} \quad (7)$$

(7) we use the Kramer method to solve a system of equations:

$$\Delta = \begin{vmatrix} m_1 + k(p-2) + l - 2 & 0 & q-p+1 \\ -1 & \beta_1 & 1 \\ \beta_2 & -1 & 1 \end{vmatrix} = 2\beta_1(m_1 + k(p-2) + l - 2) + (q-p+2) - \beta_1\cdot\beta_2\cdot(q-p+1) +$$

$$+ 2(m_1 + k(p-2) + l - 2) = 2(\beta_1 - 1)\cdot(m_1 + k(p-2) + l - 2) - (q-p+2)\cdot(1 - \beta_1\cdot\beta_2)$$

ISSN: 2181-3906 2023

International scientific journal
«MODERN SCIENCE AND RESEARCH»
VOLUME 2 / ISSUE 10 / UIF:8.2 / MODERNSCIENCE.UZ

$$\frac{d}{d\varphi}\left(f_i^{m_i-1}\left|\frac{df_i^k}{d\varphi}\right|^{p-2}\frac{df_i^l}{d\varphi}\right)+\left(\frac{N}{s}\varphi\right)^{-\frac{2s}{N}-2}\frac{d^2f_i}{d\varphi^2}-2\left(\frac{N}{s}+1\right)\left(\frac{N}{s}\varphi\right)^{-\frac{2s}{N}-3}\frac{df_i}{d\varphi}+\frac{s-1}{\varphi}f_i^{m_i-1}\left|\frac{df_i^k}{d\varphi}\right|^{p-2}\frac{df_i^l}{d\varphi}+$$

$$+\left[\mu\left(\frac{N}{s}\varphi\right)^{-\frac{ns}{N}-1}+\left(\frac{N(s-1)}{s}-2\right)\left(\frac{N}{s}\varphi\right)^{-\frac{2s}{N}-1}+2\left(\frac{N}{s}+1\right)\left(\frac{N}{s}\varphi\right)^{-\frac{2s}{N}-3}\right]\frac{df_i}{d\varphi}-\alpha_i\left(\frac{N}{s}\varphi\right)^{-\frac{ns}{N}}f_i+\gamma_i f_{3-i}^{\beta_i}=0.$$

$$a=const\geq 0$$

$$b_i=\frac{1}{\gamma}\cdot\left(\frac{1}{s}\right)^{\gamma}\cdot\left(\frac{\mu}{l\cdot k^{p-2}}\right)^{\frac{1}{p-1}}\cdot\left(\frac{m_i+k\cdot(p-2)+l-2}{p-1}\right)$$

(1) - the following theorem and lemma were obtained from the solution of the problem using the linear separation algorithm.

Lemma 1. Let $u_0\in C_0(R^1)$. Then there are constant $a_2>0$, $T>\exp(1+S(\mu-1))$

$$s>\frac{S(\mu-1)}{2(1-(1+S(\mu-1)))/\ln T}>0$$

such that the function

$$u_+(t,\varphi)=[(T+t)\ln(T+t)]^{-\frac{s}{s(\mu-1)+2}}f(\psi(\tau)\xi;a_1),$$

where $\psi(\tau)=\left(1+\frac{s}{T}\right)^{-\frac{1}{2}}$,

is the upper solution of problem (1)-(2).

Proof. It is sufficient to show that, under these constraints, function $Q_+=f(\xi\psi(\tau);a_2)$ is the upper solution of problem (13)-(14) (i.e., $D(Q_+)\leq 0$) everywhere in $(\ln T,+\infty)\times\left\{|\xi|<\frac{a_2}{\psi(\tau)}\right\}$.

Substituting function 1 in (13) we get.

$$DQ_+\equiv-\frac{A_0}{\tau}d^{\frac{1}{\mu-1}}\left\{\frac{2}{\mu-1}\left(\frac{a_2^2}{d}-1\right)F_1(\tau)-F_2(\tau)+d^{\frac{s(\mu-1)+2}{s(\mu-1)}}A_0^{\frac{s(\mu-1)+2}{s}}\right\}, \qquad (15)$$

where

$$d=\left|\left(a_2^2\right)-\frac{\xi^2}{1+\frac{s}{\tau}}\right|\in\left(0,a_2^2\right),\quad \tau>\ln T,$$

$$F_1(\tau)=\frac{s}{1+\frac{s}{\tau}}\frac{1}{4\mu s(s(\mu-1)-2)}-\frac{1}{2}\frac{\frac{s}{\tau}}{1+\frac{s}{\tau}}-\frac{s(\mu-1)}{2s(\mu-1)+4},$$

$$F_2(\tau)=\frac{s}{1+\frac{s}{\tau}}\frac{s}{s(\mu-1)+2}+\frac{s}{s(\mu-1)+2}.$$

To fulfill inequality $DQ_+\le 0$ in (10), it is necessary to fulfill condition $F_1(\tau)>0$ for all $\tau>\ln T$, which takes place under the imposed conditions on T and s, specified in the conditions of the lemma. Then, due to the uniform boundedness of τ functions $F_1(\tau)$ and $F_2(\tau)$, for all sufficiently large a_2^2, the inequality will be satisfied

$$\left[F_2(\tau)A_0^{-\frac{s(\mu-1)+2}{s}}\right]^{\frac{s(\mu-1)}{s(\mu-1)+2}}\le a_2^2\left[1+\frac{d}{2}\frac{F_2(\tau)}{F_1(\tau)}\right]^{-1},$$

which provides the implementation of inequality $DQ_+\le 0$. In turn, for large a_2, the estimate is also valid

$$Q_0(\xi)<Q_+(\ln T;\xi)=f(\psi(\ln T)\xi;a_2),$$

and hence $f(\psi(\ln T)\xi;a_2)$ is the upper solution of problem (13)-(14) (Lemma 1.). Respectively $w_+(t,\varphi)$ is the upper solution of problem (4)-(5), and $u_+(t,x)$ is the problem (1)-(2). The lemma is proved.

Theorem. For any function $u_0\in C_0(R^1)$, there are constants $a_->0$, $a_+>0$, and $T>1$ such that the solution of problem (1)-(2) in $(T,+\infty)\times R^1$ satisfies the inequality

$$[(T+t)\ln(T+t)]^{-\frac{s}{s(\mu-1)+2}}f(\xi;a_-)\le u(t,x)\le[(T+t)\ln(T+t)]^{-\frac{s}{s(\mu-1)+2}}f(\xi;a_1)$$

Proof. The validity of the theorem follows directly from Lemmas 1, where the existence of lower and upper solutions is proved and the corresponding estimates are obtained. In this case, the constant T is chosen taking into account the condition of Lemma 1 and requirement $u(T,0)>0$, the constants a_- and a_+, for example, as follows:

$$a_-=a_1,\ a_+=a_2\sqrt{1+\frac{s}{\ln T}}.$$

Then, under the conditions of the theorem, we obtain

$$f(\xi;a_-)\le Q(\tau,\xi)\le f(\xi;a_+)$$

everywhere in $(\ln T;\infty)\times R_+^1$. Then the statement of the theorem follows taking into account (13)-(14) and (4)-(5). The theorem is proved.

Results of numerical experiments and visualization

When solving the problem numerically, the equation is approximated on a grid using an implicit scheme of variable directions (for the multidimensional case) in combination with the balance method. Iterative processes were built on the basis of the Picard, Newton method as well as special method.

A computational algorithm has been developed. When developing software that illustrates the simulation process of solution behavior over time (visualization), the Visual Studio 2019 C# environment was used with the inclusion of the Open GL graphics library.

The results of computational experiments show that all iterative methods are suitable for the constructed scheme. To achieve the same accuracy, the Newton method requires fewer iterations than the Picard method. In some cases, the number of iterations is almost twice, and the maximum number of iterations is 3-4 times less than in other methods. Since power-law nonlinearity is present in the right-hand side of equation (1), naturally the special method gives better results than the Picard method.

As a test example, we used solutions of equation (1) obtained by the methods of reference equations and nonlinear splitting [2]. Figures 1-4 show the calculation results for various values of parameters $p, n, m, k, l, \mu, \beta$ and time.

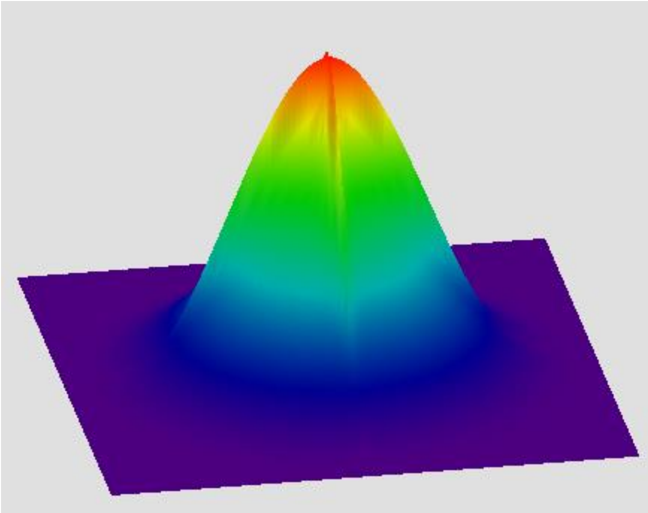

Fig.3. p=2.1, n=1.2, m=1, k=1, l=1

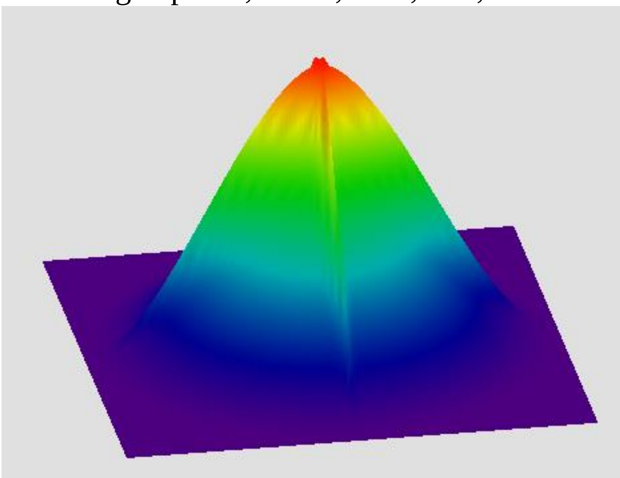

Fig.4. p=2.3, n=1, m=1.2, k=1.3, l=1

Conclusion

The results of computational experiments show that all of the listed iterative methods are effective for solving nonlinear problems and lead to nonlinear effects if we use self-similar solutions constructed by the nonlinear splitting method and the standard equation method as the initial approximation of the solution [4, 6].

As expected, in order to achieve an identical accuracy, the Newton method requires fewer iterations than the methods of Picard and a special method because of the successful choice of the initial approximation. Note that in each of the cases considered, the Newton method has the best convergence due to the choice of a good initial approximation.

In some cases, the total iteration amount is almost two times and the maximum iteration is almost 4 times less than other methods.

The results of numerical calculations show the effect of the finite velocity of disturbance propagation and the localization of the solution depends on the values of the numerical parameters.

All results of numerical experiments are presented in the form of visualizedanimation.

Acknowledgements

We are very grateful to experts for their appropriate and constructive suggestions to improve this template.

REFERENCES

1. Angar Jungel. Cross-Diffusion systems with entropy structure. arXiv: 1710.01623v1 [math.AP] 4 Oct 2017. Proceedings of EQUADIFF, (2017).
2. Aripov M., Sadullaeva Sh. Properties of solutions to reaction-diffusion equation with double nonlinearity with distributed parameters. Log SFU. Ser. Mat and Phys., 6: 2, 157–167 (2013).
3. Ж. М. Таштемиров, А. Т. Хайдаров Влияние источника тепла на плотность окружающей среды в нелинейных процессах тепловыделения в двухмерных областях (pp. 295-299) SCIENTIFIC PROGRESS VOLUME 4 I ISSUE 2 I 2023 _ISSN: 2181-1601.
4. Aripov M., Abdullaeva Z. On the bottom of the exact solution of a nonlin- ear problem with absorption or a source. Bulletin of the TATU, No.4, 107–113 (2016).
5. Khaidarov A. T, Toshtemirov J. M. HEAT SOURCE DENSITY IN NON-LINEAR HEAT DISSIPATION PROCESSES //Proceedings of Scientific Conference on Multidisciplinary Studies. – 2023. – Т. 2. – №. 10.–С. 72-80.
6. Khaidarov A. T., Toshtemirov J. M. MODELING OF THE DEPENDENCE OF THE CONDUCTIVITY OF THE NON-ELECTRIC MEDIUM OF IRON METAL //Proceedings of Scientific Conference on Multidisciplinary Studies. – 2023. – Т. 2. – №. 10. – С. 88-89.
7. Mamatov A. U., Boboqandov M. M. To investigate the effect of nonlinear environmental speed on nonlinear heat dissipation process in multidimensional areas by the method of variable directions. Journal of HIGHER SCHOOL, ISSN 2409-1677, No.12,34-38(2019).
8. Rakhmonov Z. R., Urunbayev J. E. On a problem of cross-diffusion with nonlocal boundary conditions. J. Sib.Fed. Univ. - Math. Phys., 2019.
9. Mamatov A. U. Rikhsiboev D. R. Investigation of the ambient density and the state of thermal heating by the method of variable directions in the process of uneven heat distribution. Journal of HIGHER SCHOOL, ISSN 2409-1677, No. 12,39-41 (2019).
10. Mamatov A. U. Numerical simulation of the heat conduction process with double nonlinearity in the two-dimensional case in the presence of transfer and source (absorption). Scientific Journal of Samarkand University, ISSN: 2091-544, No. 1, 41-44 (2020).
11. M. Aripov and S. Sadullaeva 2015. An asymptotic analysis of a self-similar solution for the double nonlinear reaction-diffusion system. Nanosyst. Physics, Chem. Math.
12. Martinson L.K. Evolution of a heat pulse in a nonlinear medium with volumetric heat absorption. TVT, Vol. 21, No.4, 801–803 (1983).

13. Samarsky A.A., Elenin G.G., Zmitrenko N.V., Kurdyumov S.P. and Mikhailov A.P. Gorenje nonlinear environment in the form of complex structures. DAN USSR, 1977 Vol. 237, No.6, 1330–1333 (1977).
14. Martinson L.K., Pavlov K.B. On the spatial localization of thermal disturbances in the theory of nonlinear thermal conductivity. ZhVM and MF, Vol. 12, No.4, 1048–1053 (1972).
15. Khasanov, A. A. (2017). Methods and methods of forming economic education through interdisciplinary communication through information technology. Journal, (3), 38.
16. Sharipov, D., Abdukadirov, A., Khasanov, A., & Khafizov, O. (2020, November). Mathematical model for optimal siting of the industrial plants. In 2020 International Conference on Information Science and Communications Technologies (ICISCT) (pp. 1-3). IEEE.
17. Safarov I.I1 , Almuratov Sh.N2 , Teshaev M.Kh3. On the dynamic stress-strain state of isotropic rectangular plates on an elastic base under vibration loads. Indian Journal of Engineering, 127-133.
18. NK Esanov, SN Almuratov, US Jurayev. Sayoz o ‘rnatilgan uch qatlamli sferik qobiqlarning erkin tebranishi. Nazariy va amaliy tadqiqotlar xalqaro jurnali 2 (2), 51-56.
19. II Safarov, NR Kulmuratov, MR Ishmamatov, NB Axmedov, S Almuratov. ON THE DYNAMIC STRESSED-DEFORMED STATE OF ISOTROPIC RECTANGULAR PLATES ON AN ELASTIC BASE WITH VIBRATION LOADS. Theoretical & Applied Science, 362-367.
20. Mamatov A.U. Modeling the effect of the two-fold non-linear heat dissipation equation on biological population with ambient density, Scientific Journal of Samarkand University.